Young's double-slit experiment optimizer: A novel metaheuristic optimization algorithm for global and constraint optimization problems

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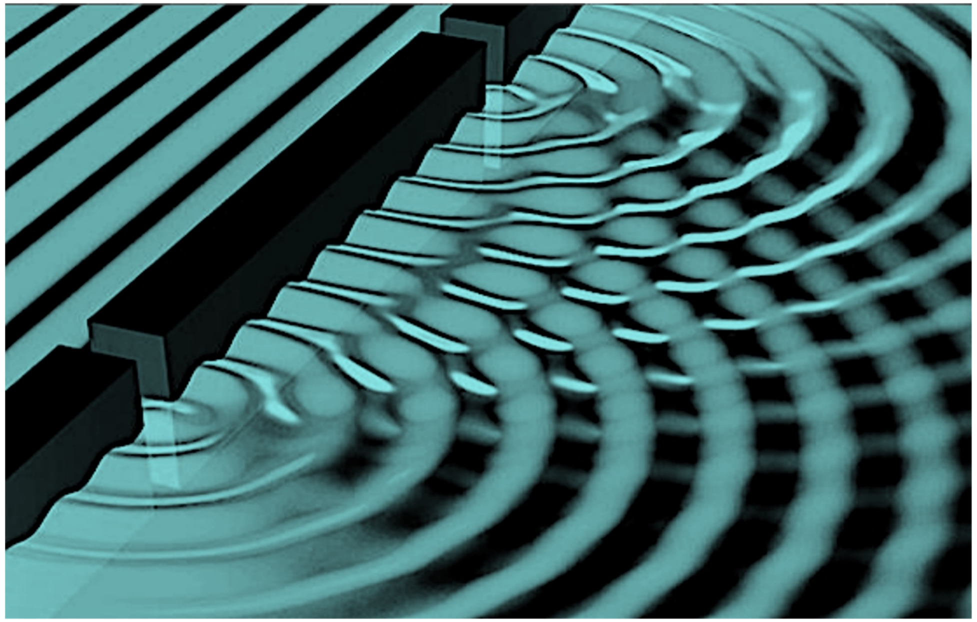
Inspiration

- Young's Double-Slit Experiment (YDSE) optimizer is a population-based metaheuristic algorithm that simulates one of the most well-known classical experiments in the world of physics.

- The Young's experiment demonstrates the wave nature of light.



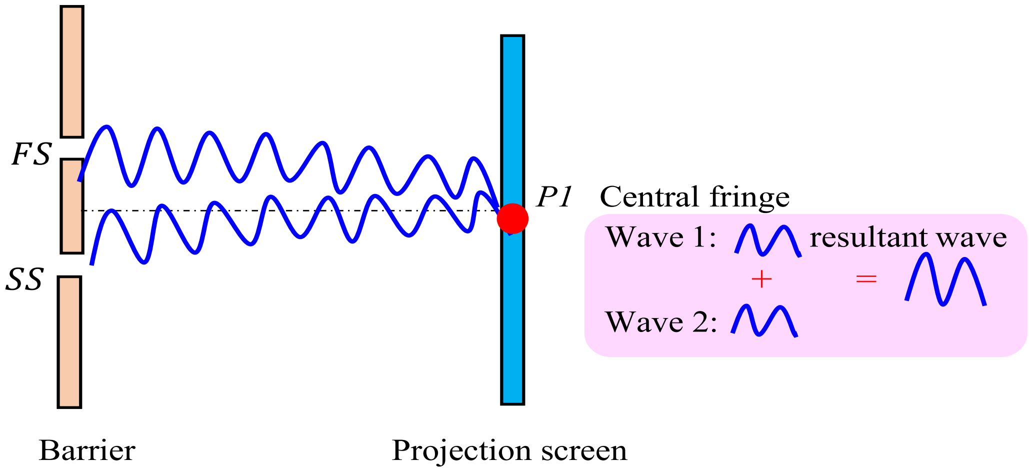
Water and light having wave-like pattern

Interference

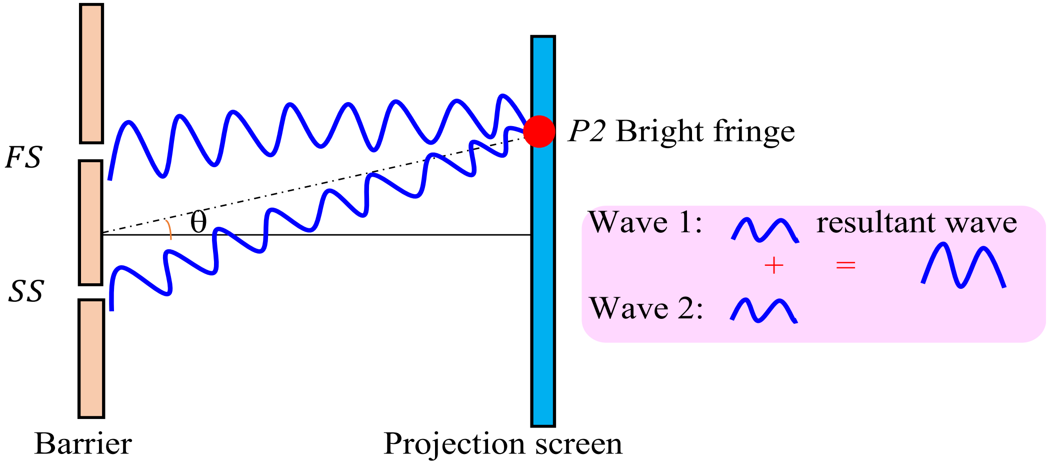
- Interference occurs when two waves meet while traveling along the same medium. Some conditions must be met for the wave interference in YDSE:
  - The light source must be monochromatic.
  - The waves must be of the same frequency.
  - The direction of the waves must be the same.
  - The amplitudes of the two waves must be equal.
  - The openings of the two slits must be thin.

Constructive interference

- Constructive Interference (CI) occurs when the two waves travelling the path from each slit arrive in phase at a point on the screen.



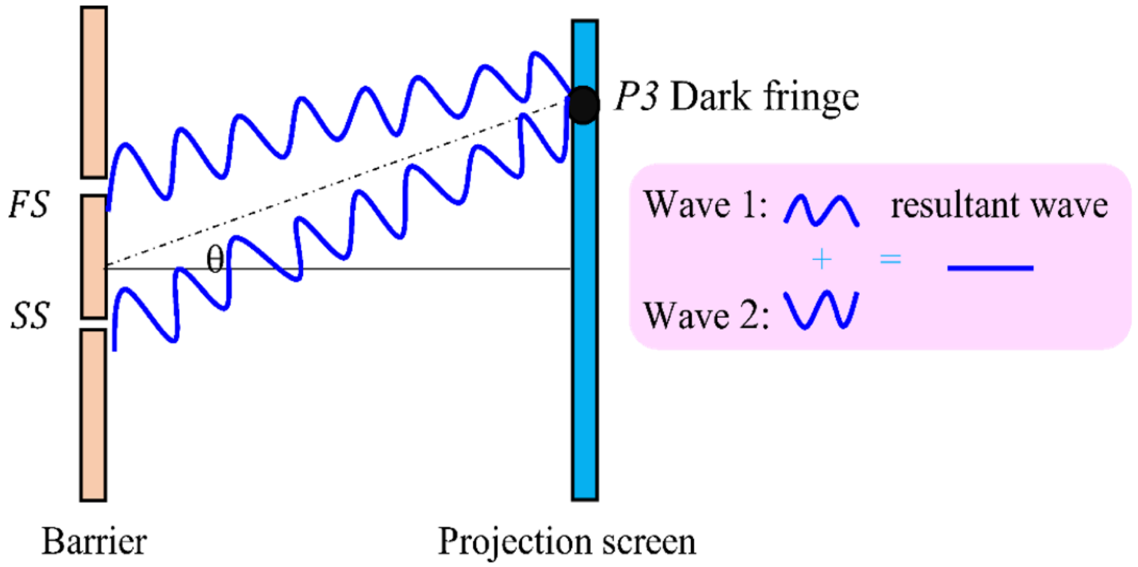
Constructive interference of central fringe



Constructive interference of bright fringe

Destructive interfernce

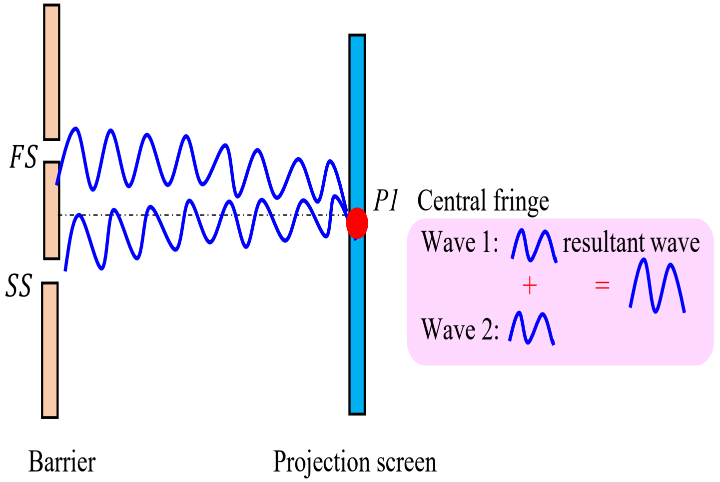
- Destructive Interference (DI) occurs when the two waves arrive out of phase at the screen (crest to trough).
- From the figure, in the DI, the two interfering waves cancel each other leading to a lower amplitude.



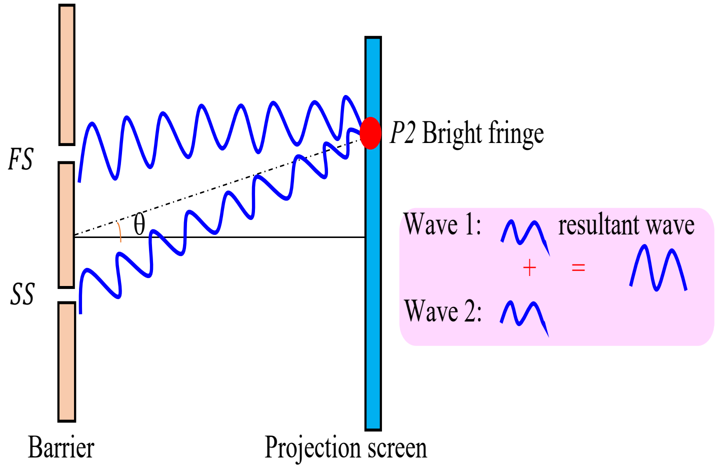
Destructive interference of dark fringe

Types of fringes

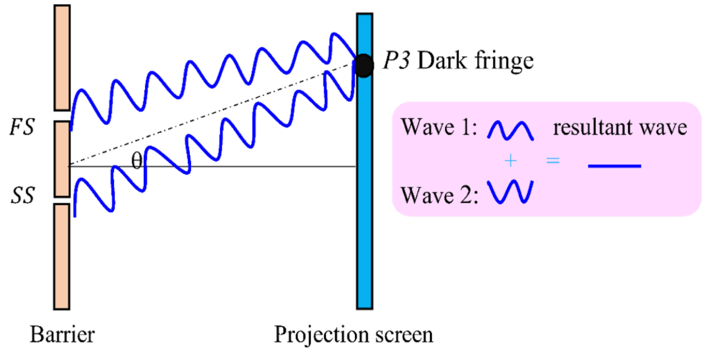
Central fringe



Bright fringe



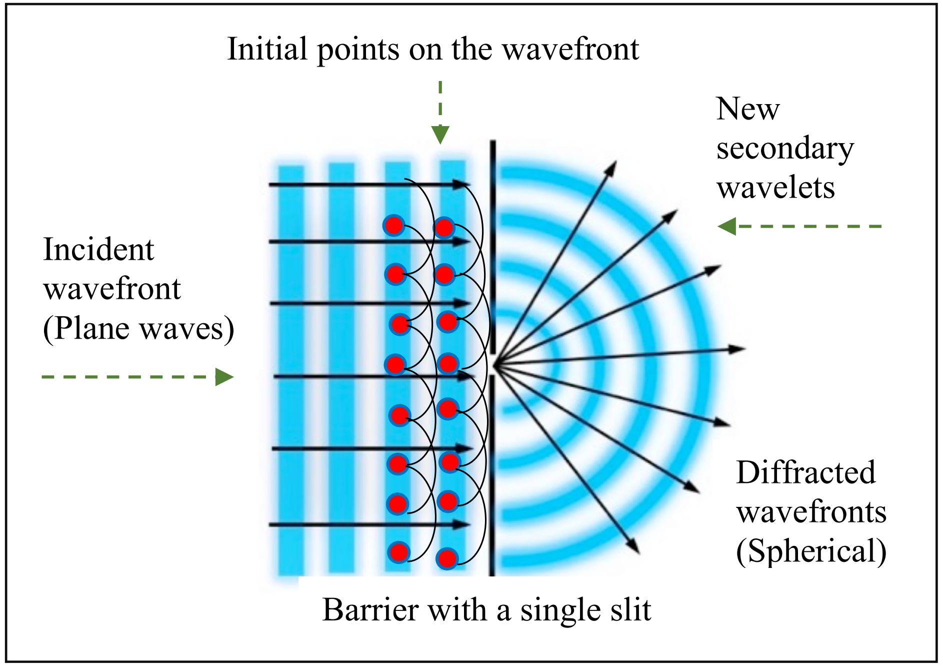
Dark fringe



Hugens' principle

- According to Huygens' principle, every point on the wavefront will act as a secondary source and will be able to emit new secondary waves in all directions.

- These new secondary waves operate effectively in the forward direction with the speed of the wave.

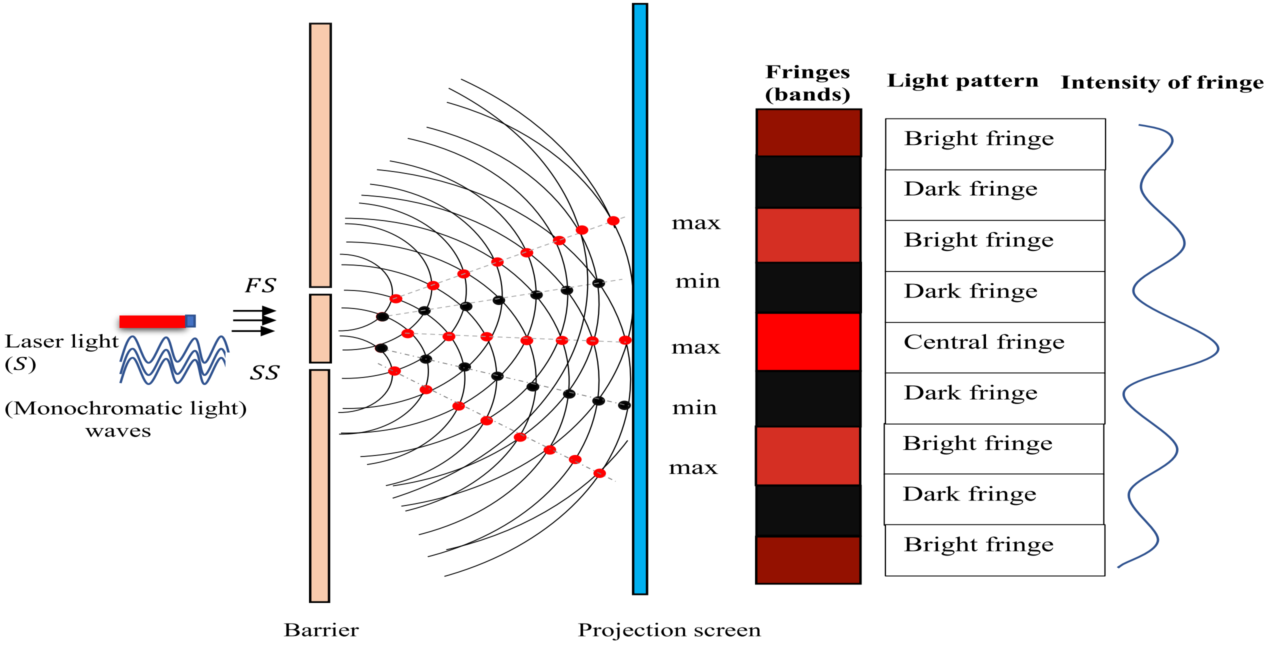


Huygens' principle

YDSE Optimizer

- In the experiment, a source of monochromatic light waves (*S*) is first projected into a barrier with two closely spaced slits.

- An initial population (*S*) of *NP* monochromatic light waves are created. Each monochromatic wave has a dimension of *Dim*.



Huygens' principle application

- In YDSE optimizer, the monochromatic waves spread out in all directions from the two slits according to Huygens' principle. Each wavefront point behaves as a source and a center of a new wave.
- For simplicity, the number of points on the wavefront emerging from the slits is equal to NP and calculated from S as:

$$FS_i = S_i + L \times rand1(-1,1) \times (S_{mean} - S_i), \quad i = 1,\ 2,\ \ldots,\ NP$$

$$SS_i = S_i - L \times rand2(-1,1) \times (S_{mean} - S_i), \quad i = 1,\ 2,\ \ldots,\ NP$$

$$S_{mean} = \frac{1}{NP} \sum_{i=1}^{NP} S_i$$

- S_{best} is the best individual in population S. S_{mean} is the mean of the current population S. FS_i is the point i created on the wavefront outgoing from the first slit FS. SS_i is the point i created on the wavefront outgoing from the second slit SS.

Path difference

- In this step, the outgoing waves from the two slits (FS and SS) did not travel the same distance.
- Path difference occurs when one wave from one slit may travel a distance that can be larger than, smaller than or equal to the other wave to reach a point on the screen and simulated by:

$$X_i = \left(\frac{FS_i + SS_i}{2}\right) + \Delta L$$

$$\Delta L = \begin{cases} 0 & \text{if } CI \text{ occurs at } m = 0 \\ (2m+1)\frac{\lambda}{2} & \text{if } DI \text{ occurs at odd } m \\ m\lambda & \text{if } CI \text{ occurs at even } m \end{cases}$$

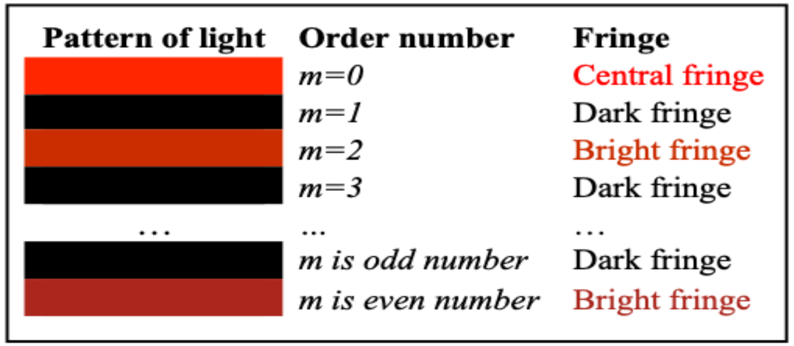
- ΔL is the path difference between FS_i and SS_i.

Path difference (cont.)

- In YDSE, the points of constructive and destructive interference result in bright and dark fringes on the projection screen that has an order number (m).
- The population can be viewed as a set of fringes resulting from DI and CI, in which bright and dark fringes are arranged in the population such that:
  - The central fringe takes a zero-order number.
  - The bright fringe takes an even order number.
  - The dark fringe takes an odd order number.

Index mapping of wavefront points into order number (m)

Index (i)	Order number (m)	Type of Fringe
1	*0*	Central
2	*1*	Dark
3	*2*	Bright
4	*3*	Dark
⋮	⋮	⋮
NP	*NP-1*	…

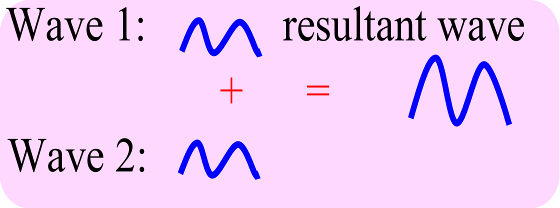


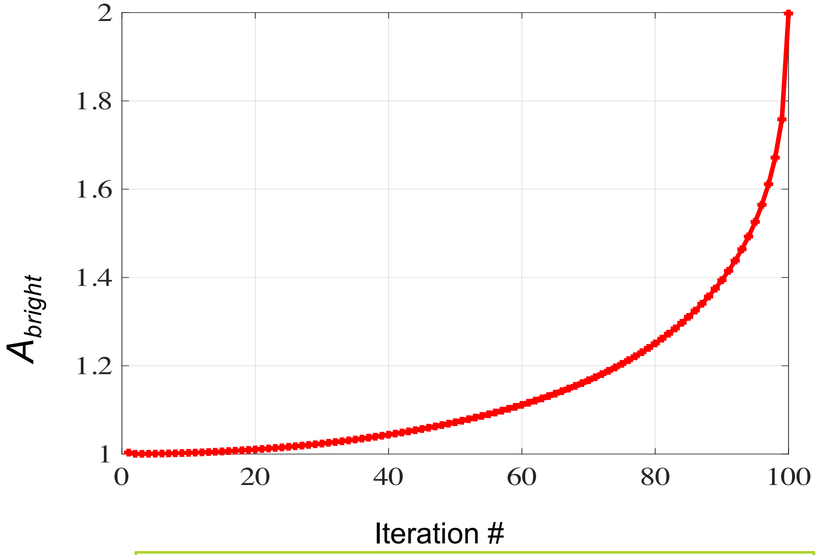
Wave amplitude for constructive interference

- In CI, the resulting wave from the two interfering waves has a higher amplitude than the previous waves. This behavior is simulated using:

$$A_{bright}^{t+1} = \frac{2}{1+\sqrt{|1-\beta^2|}}, \quad \beta = \frac{t}{T}\cosh(\pi / t)$$

- A_{bright}^{t+1} is the average amplitude of the wave at bright fringe at iteration *t+1*. *T* defines the maximum number of iterations. *cosh* is a hyberpolic function.
- It is assumed that the average amplitude increases and takes the value from 1 at the first iteration to 2 in the last iteration.





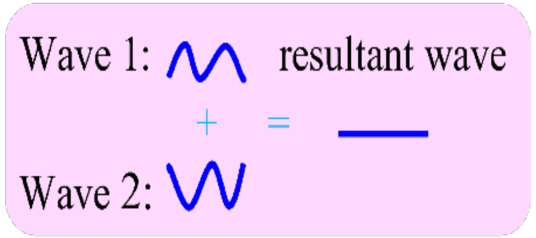
Average of wave amplitude of bright fringe versus iterations

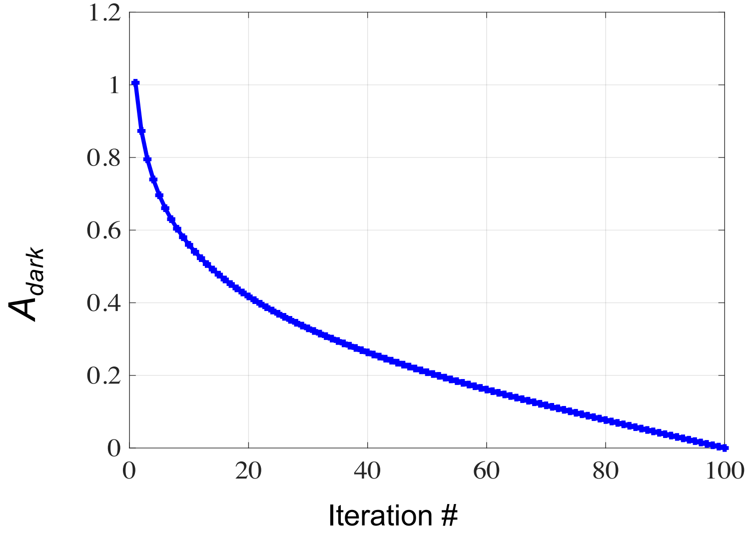
Wave amplitude for destructive interference

- In DI, the resulting wave from the two interfering waves has a lower amplitude than the previous waves. This behavior is simulated using.

$$A_{dark}^{t+1} = \delta \times \tanh^{-1}(-\frac{t}{T}+1)$$

- A_{dark}^{t+1} is the average amplitude of the wave at dark fringe at iteration *t+1*. δ is a constant value =0.38.
- It is assumed that the average amplitude decreases with time and takes the value from 1 at the first iteration to 0 in the last iteration .





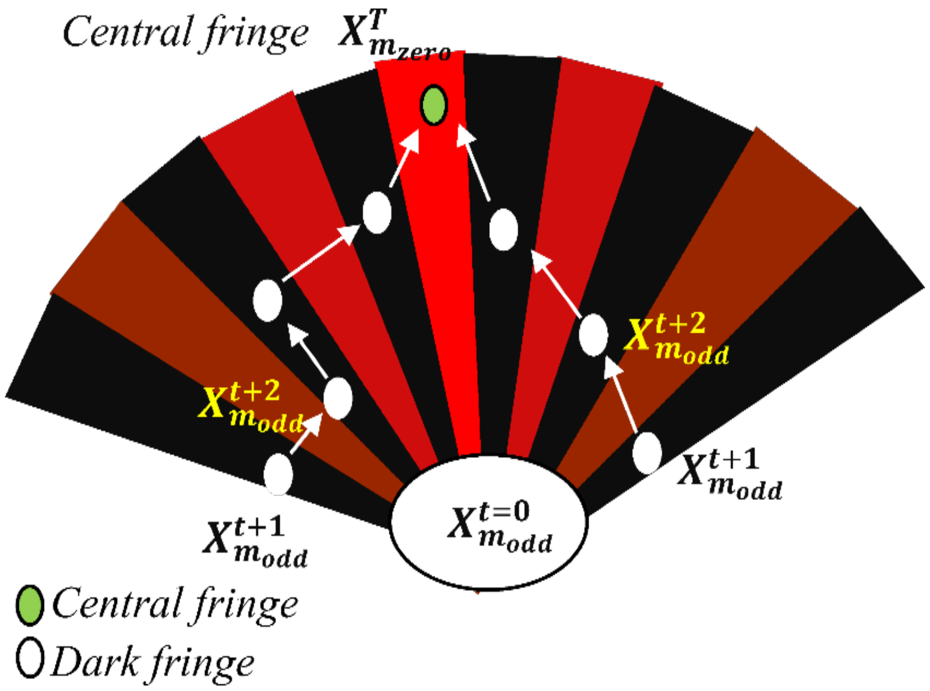
Average of wave amplitude of dark fringe versus iterations

Interference

- The search space is divided into three regions: dark regions representing dark fringes; bright regions representing light fringes; and one central region representing central bright fringe.
- In YDSE optimizer, the order number is divided by three particles: even, odd, and 0.
  - ✓ If a fringe has an even order number, it falls in the bright region.
  - ✓ If a fringe has an odd order number, it falls in the dark region.
  - ✓ If a fringe has zero, it falls in the central bright region.
- This behavior perfectly mimics YDSE to determine locations of fringes.

Exploration (destructive interference)

- During the optimization process, the solution moves in search space based on its order number. If it has an odd order number, it moves in the dark regions towards the central bright region that is expected to contain the optimal solution.
- The solutions in the dark areas are expected to have lower fitness values than those in the bright areas.
- Following the dark regions in the search and leaving the search around the current best fringe helps to explore other different regions of the search space.



The behavior of solution convergence in the search space for dark fringe

Exploration (destructive interference)

- The update strategy for the dark fringe resulting from destructive interference is:

$$X_{m_{odd}}^{t+1} = X_{m_{odd}}^{t} - (r_1 \times A_{dark}^{t+1} \times Int_{m_{odd}}^{t+1} \times X_{m_{odd}}^{t} - z \times X_{best}^{t})$$

$$Int_{m_{odd}}^{t+1} = Int_{\max}^{t+1} \times \cos^2(\frac{\pi d}{\lambda L} y_{dark}^{t+1})$$

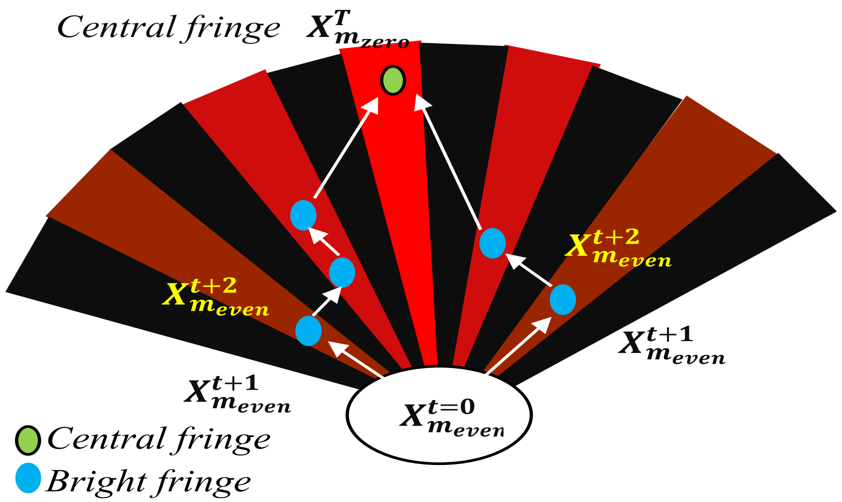
$$Z = \frac{a}{H}$$

$$a = t^{2 \times r_2 - 1}$$

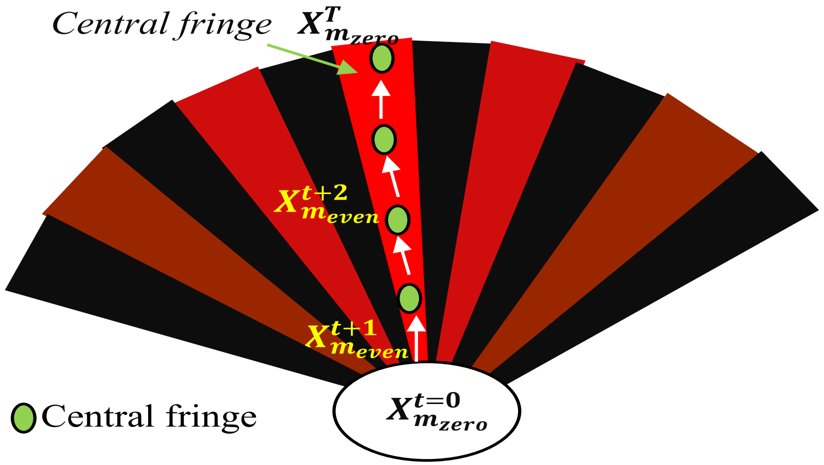
- $X_{m_{odd}}^{t+1}$ is the new m_{odd}^{th} dark fringe at iteration *t+1*. $X_{m_{odd}}^{t}$ is the old m_{odd}^{th} dark fringe at iteration *t*. $Int_{m_{odd}}^{t+1}$ indicates the light intensity of the m_{odd}^{th} dark fringe at iteration *t+1*. Z is a trial vector defined each iteration to search around the current best obtained fringe, hoping to find the optimal solution. a takes a random value in $[T^{-1}, T]$. H is a random vector defined in [-1,1]. X_{best}^{t} refers to the best-obtained solution in the iteration (t). r_1 and r_2 are random values belong to [0,1]. $Int_{\max}^{t+1}$ represents the maximum intensity detected in the central region in iteration *t+1*. y_{dark}^{t+1} measures the distance between the central fringe and the m_{odd}^{th} dark fringe.

Exploitation(Constructive interfernce)

- In the exploitation phase, the algorithm exploits the promising areas in bright fringe areas, which are assumed to contain the optimum. YDSE optimizer works to exploit all the promising areas in bright fringe regions.



The behavior of solution convergence in the search space for bright fringe



The behavior of solution convergence in the search space for central bright fringe

Exploitation(Constructive interfernce)

- The bright regions is updated using constructive interference as:

$$X_{m_{even}}^{t+1} = X_{m_{even}}^{t} - ((1-g)\times A_{bright}^{t+1} \times Int_{m_{even}}^{t+1} \times X_{m_{even}}^{t} + g\times(Y))$$

$$Y = X_{m_{rand1}}^{t} - X_{m_{rand2}}^{t}$$

$$Int_{m_{even}}^{t+1} = Int_{\max}^{t+1} \times \cos^2(\frac{\pi d}{\lambda L} y_{bright}^{t+1})$$

- $X_{m_{even}}^{t+1}$ is the new m_{even}^{th} bright fringe at iteration *t+1*. $X_{m_{even}}^{t}$ is the current bright fringe at iteration *t*. Y represents the difference between two randomly selected fringes which may be bright, dark or both. $Int_{m_{even}}^{t+1}$ is the intensity of the m_{even}^{th} bright fringe at iteration *t+1*. y_{bright}^{t+1} measures the distance between the central fringe and the m_{even}^{th} bright fringe. $X_{r_b}^{t}$ is a bright fringe selected randomly from the population.

Exploitation(Constructive interfernce)

- The update strategy in the central region is:

$$X_{m_{zero}}^{t+1} = X_{best}^{t} + (A_{bright}^{t+1} \times Int_{\max}^{t+1} \times X_{m_{zero}}^{t} - r_3 \times z \times X_{r_b}^{t})$$

$$Int_{\max}^{t+1} = C \times q$$

$$q = \frac{t}{T}$$

- $X_{m_{zero}}^{t+1}$ is the new central fringe which has an order number zero at iteration *t+1.*
- $X_{m_{zero}}^{t}$ is the current central fringe at iteration *t*. r_3 is a random number belonging to [0,1] and raised to power of 5. $Int_{\max}^{t+1}$ is the maximum intensity detected by the central fringe at iteration *t+1*.
- q is an increasing parameter during iterations from zero to one. C indicates a constant value equal to 10^{-20} to indicate the maximum intensity value in the last iteration.

Exploitation(Constructive interfernce)

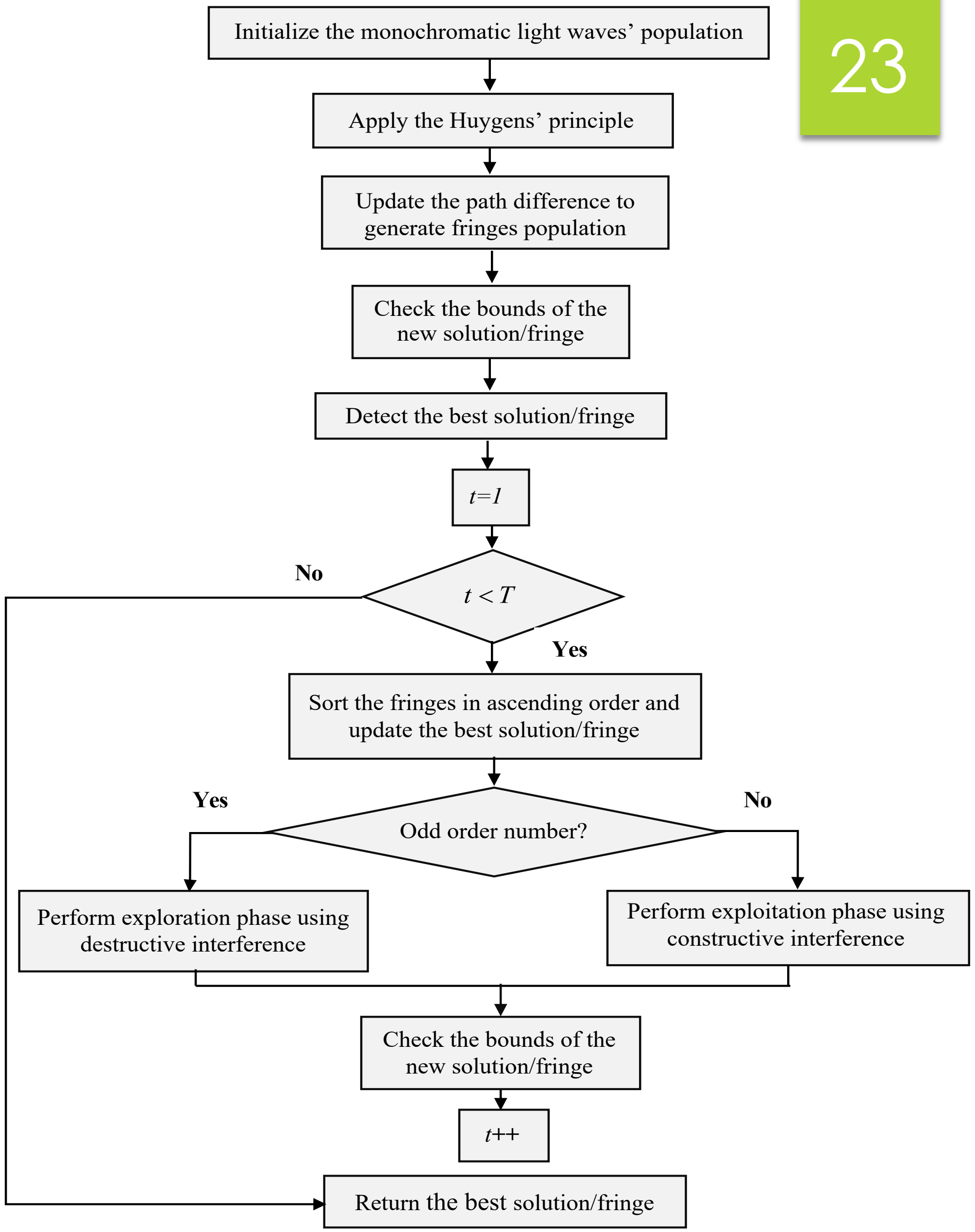
- According to YDSE, the global optimum is a solution that has the maximum intensity value.
- In the first iterations, the central fringe is far away from the global optimum and hence the central fringe has a lower intensity value.
- During the iterations, the central fringe gets closer to the global optimum and consequently, the intensity is increased over iterations.

Pseudocode of the YDSE optimizer

Algorithm 1 YDSE optimizer

```
Initialize a monochromatic source of NP light waves
Apply Huygens' principle
Update the path difference and generate the population of light fringes
Check the bounds of the fringes
Evaluate the fitness of the solutions/fringes
Detect the best fringe with minimum fitness
Assign an order number for each solution/fringe
Define the maximum intensity for the solution /fringe in the central region
Define the current iteration t=1
while (t < T)
    Update q parameter
    Sort the fringes from the best to the worst based on their fitness
    Update the best fringe
    for m=0:NP-1
        Update Z vector
        if ( m = 0 )  /* Exploitation phase */
            Update intensity and amplitude for the central fringe
            Update the central fringe X^{t+1}_{m_zero} using constructive interference
        else if (m = even number)
            Update intensity and amplitude for the m^{th}_{even} bright fringe
            Update the bright fringe X^{t+1}_{m_even} using constructive interference
        else    /* Exploration phase */
            Update intensity and amplitude for the m^{th}_{odd} dark fringe
            Update the dark fringe X^{t+1}_{m_odd} using destructive interference
        end
        Check the bounds of each fringe
    end for
    Update the current number of iteration t by t=t+1
end while
return the best fringe
```

Flowchart of the YDSE optimizer



Time complexity of YDSE

$$
\begin{aligned}
&\textit{Time complexity (YDSE)} = O(\textit{initilization}) + O(\textit{Huygens' principle}) + \\
&O(\textit{path difference update}) + O(\textit{evaluation}) + O(\textit{solutions updating}) \\
&= O(NP \times Dim) + O(2 \times NP) + O(NP) + O(C \times NP) + O(T \times C \times NP) \\
&\quad + O(\frac{1}{2} \times T \times NP \times Dim) + O(\frac{1}{2} \times T \times NP \times Dim) \\
&= O(T \times C \times NP) + O(T \times NP \times Dim) \\
&= O(T \times C \times NP + T \times NP \times Dim).
\end{aligned}
$$